Institute of Actuaries of India
ACET MAY 2016
Mathematics

1. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them then $\vec{a} + \vec{b}$ is a unit vector if

   A. $\theta = \frac{\pi}{3}$

   B. $\theta = \frac{\pi}{2}$

   C. $\theta = \frac{\pi}{4}$

   D. $\theta = \frac{2\pi}{3}$.   1 mark

2. If the position vectors of the two points A and B are $\vec{i} + 2\vec{j} - 3\vec{k}$ and $2\vec{i} - 4\vec{j} + \vec{k}$ respectively , then $|\overrightarrow{AB}|$ is

   A. $\sqrt{53}$

   B. 53

   C. 1

   D. 6.   2 marks

3. Fill in the blank: A unique polynomial of degree __ passes through $n + 1$ given points on the plane (no pair of points lying above one another).

   A. $n + 1$

   B. $n + 1$ or less

   C. n

   D. n or less.   1 mark

4. The natural domain of the real function f defined by $f(x) = \sqrt{x - 1}$ is

   A. $[0, \infty)$

   B. $(-\infty, \infty)$

   C. $[1, \infty)$

   D. $(-\infty, 1]$.   1 mark

5. If $f(x) = x + 1$ and $g(x) = x^2$, then $f \circ g(3)$ is

   A. 4

   B. 10

   C. 16

   D. 9.   1 mark

6. The McLaurin series expansion for $\frac{1}{1+x}$ is

    A. $1 + x + x^2 + \cdots$

    B. $x + x^2 + x^3 + \cdots$

    C. $1 + x + 2x^2 + 3x^3 + \cdots$

    D. $1 - x + x^2 - x^3 + \cdots$.   1 mark

7. $\frac{1}{(x-1)(x+1)}$ is resolved into partial fractions as

    A. $\frac{1}{(x-1)} + \frac{1}{(x+1)}$

    B. $\frac{1}{(x-1)} - \frac{1}{(x+1)}$

    C. $\frac{1}{2(x-1)} + \frac{1}{2(x+1)}$

    D. $\frac{1}{2(x-1)} - \frac{1}{2(x+1)}$.   1 mark

8. The roots of the quadratic polynomial $2x^2 + 7x - 4 = 0$ are

    A. $-4, \frac{1}{2}$

    B. $\ \ 4, \ \frac{1}{2}$

    C. $-4, -\frac{1}{2}$

    D. $\ \ 4, \ -\frac{1}{2}$.   2 marks

9. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{3x}$ is

    A. e

    B. e^3

    C. e^{-3}

    D. $3e$.   1 mark

10. Let $y = x^{-\frac{1}{2}} + \log_5 x + 6$. Then $\frac{dy}{dx}$ is

    A. $\frac{1}{2}x^{-\frac{3}{2}} + \frac{1}{x}$

    B. $-\frac{1}{2}x^{-\frac{3}{2}} + \frac{1}{x}\log_5 e$

    C. $-\frac{1}{2}x^{-\frac{3}{2}} + \frac{1}{x}\log_e 5$

    D. $-\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{x}\log_5 e$.   2 marks

11. The interval in which the function $f(x) = x^3 - 3x + 1$ is decreasing in x is

A. $(\infty, 1)$
B. $(1, \infty)$
C. $(-\infty, \infty)$
D. $(-1,1)$.   2 marks

12. If $f(x)$ is an even function, then for $a > 0$, $\int_{-a}^{a} f(x)\, dx$ is equal to

A. 0
B. $\int_{a}^{-a} f(x)\, dx$
C. $2 \int_{0}^{a} f(x)\, dx$
D. $-2 \int_{-a}^{0} f(x)\, dx$.   1 mark

13. The value of $\int_{0}^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\, dx$ is

A. $\pi/2$
B. $\pi/3$
C. π
D. $\pi/4$.   3 marks

14. If A is a square matrix of order 3 such that $A^2 + I = 0$ then a solution for A is

A. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} -i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & -i \end{bmatrix}$

C. $\begin{bmatrix} i^2 & 0 & 0 \\ 0 & i^2 & 0 \\ 0 & 0 & i^2 \end{bmatrix}$

D. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ 2 marks

15. If $A = \begin{bmatrix} 2 & 0 & 1 \end{bmatrix}$ then the rank of AA^T is

A. 3
B. 0
C. 1
D. 2.   1 mark

16. The table below gives the growth of a plant (y) in mm on different days (x).

x	1	3	5
y	0	4	9

Assuming linear relationship between x and y in between the observed values, the interpolated value of y when $x = 4$ is

A. 5 mm
B. 6 mm
C. 6.5 mm
D. 7.5 mm.   1 mark

17. The value of $\left(1 + \sqrt{5}\right)^3 + \left(1 - \sqrt{5}\right)^3$ is

A. 150
B. 32
C. 152
D. 30.   1 mark

18. Mr. A starts repaying a loan with the first monthly installment of Rs. 1000 and he increases the subsequent installments by Rs.50 every month. The amount he pays in the 30th installment is

A. Rs. 1500
B. Rs. 2500
C. Rs. 3000
D. Rs. 2450   1 mark

Statistics

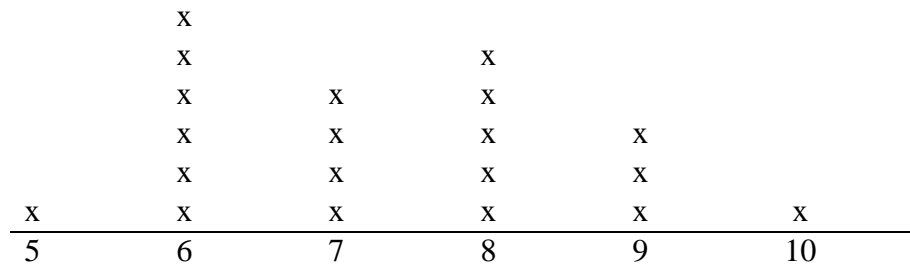19. If $\binom{n}{6} = \binom{n}{4}$ then $\binom{12}{n}$ is

    A. 66
    B. 220
    C. 24
    D. 12   1 mark

20. The shoe sizes of 20 students are plotted in the following dot diagram

```
          X
          X                    X
          X         X          X
          X         X          X          X
          X         X          X          X
X         X         X          X          X          X
5         6         7          8          9          10
```

    A. The mean shoe size is smaller than the median but larger than the mode
    B. The mean shoe size is larger than the median but smaller than the mode
    C. The mode of the shoe sizes is smaller than the median and the mean
    D. The mean shoe size is smaller than the median and the mode.   3 marks

21. The standard deviation for the population of numbers

       5   10 -12 17 -14 0 -6

    is equal to

    A. 9
    B. 13
    C. 11.5
    D. 18.   1 mark

22. A company producing electric relays has three manufacturing units producing respectively 50, 30 and 20 percent of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are, respectively, 0.02, 0.05 and 0.01. If a relay is selected at random from the output of the company, the probability that it is defective is

    A. 0.5
    B. 0.08
    C. 0.27
    D. 0.027.   1 mark

23. The events A and B are such that $P(A \cup B) = \frac{2}{3}$ and $P(A \cap B^c) = \frac{1}{3}$. Then $P(B)$ is

   A. 1
   B. 1/9
   C. 1/3
   D. 0.   1 mark

24. A random variable X has probability density function

$$f(x) = \begin{cases} kx(1-x), & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

   The value of k is

   A. 2
   B. 6
   C. 5
   D. 4.   2 marks

25. A random variable X has exponential distribution with probability density function

$$f(x) = \mu e^{-\mu x}, \qquad x, \mu > 0.$$

   The coefficient of variation of X is

   A. $1/\mu$
   B. 1
   C. μ
   D. $\sqrt{2}$.   2 marks

26. The random variable X has Poisson distribution with $P(X = 0) = 2P(X = 1)$. The variance of this distribution is

   A. 1
   B. 1/4
   C. 1/2
   D. $1/\sqrt{2}$.   2 marks
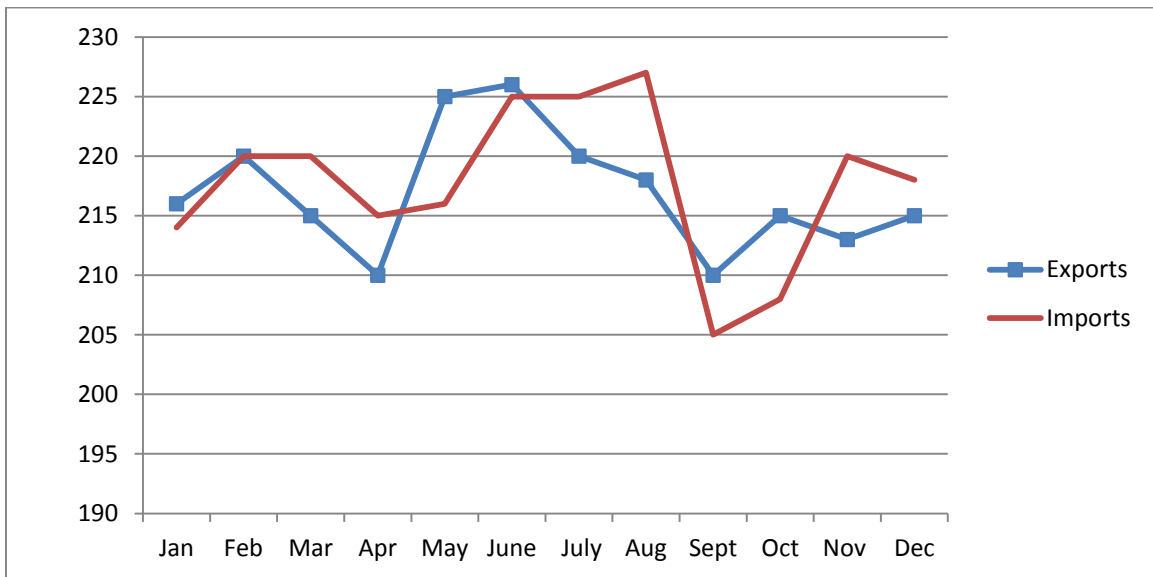
27. Which of the following statements is *not* true?

   A. The covariance between X and Y is zero implies the correlation coefficient is zero.
   B. The correlation coefficient between between X and Y lies in -1 and $+1$
   C. The correlation coefficient between X and Y is not affected by change of origin and scale.
   D. The covariance between X and Y is zero implies the random variables are independent.   1 mark

28. For a bivariate data on X and $Y,$ it is given that the correlation coefficient is $\frac{2}{3}$, the variances of X and Y respectively are 4 and 144. The regression coefficient of X on Y is

   A. 1/9
   B. 4
   C. 1/3
   D. 2.   2 marks

Data Interpretation

Study the following graph of monthly exports and imports (in Rs. million) in a particular year, given below, and note the information given in it to answer the Questions 29 to 33.

29. The maximum per cent fall in exports compared to the previous month was exhibited in the month of

   A. May
   B. June
   C. July
   D. September.   2 marks

30. In how many months was there a fall in exports, but a rise in imports compared to the previous month?

   A. 1
   B. 2
   C. 3
   D. 4.   1 mark

31. Trade balance for the whole period shown in the graph (in Rs. million) was approximately.

    A. 15
    B. −10
    C. −25
    D. 10.   3 marks

32. Fill in the blank: The highest import was about ..... times the lowest import.
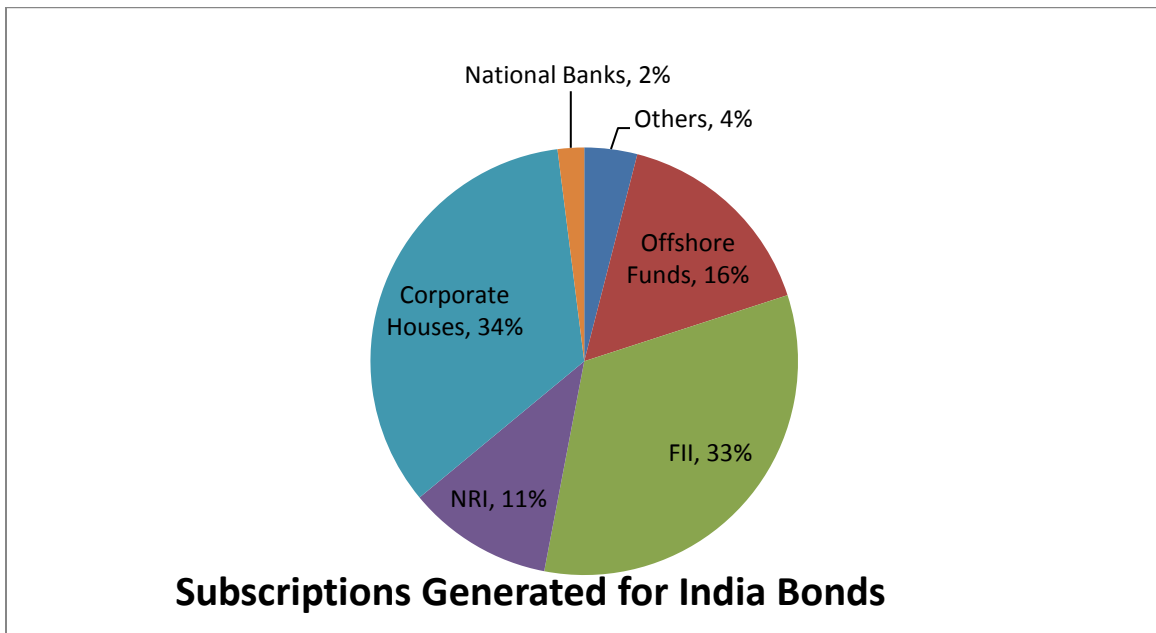
    A. 3.1
    B. 1.1
    C. 5.1
    D. 7.1.   1 mark

33. In which of the following months was the largest deficit exhibited?

    A. March
    B. July
    C. August
    D. November.   1 mark

The following pie chart shows the amount of subscriptions generated for India Bonds from different categories of investors. Use the information given in it to answer Questions 34-38.

National Banks, 2%
Others, 4%
Offshore Funds, 16%
Corporate Houses, 34%
FII, 33%
NRI, 11%

Subscriptions Generated for India Bonds

34. If the investments by Offshore Funds are Rs. 5820 crores, then the investment by corporate houses and FIIs together is approximately

    A. 24000 crores
    B. 24400 crores
    C. 25400 crores
    D. 25600 crores.   1 mark

35. What percentage of the total investment is coming from either National Banks or NRIs?

    A. 33
    B. 11
    C. 13
    D. 22.   1 mark

36. What is the ratio of investment flows into India Bonds from National Banks to Corporate houses?

    A. 1:4
    B. 1:3
    C. 1:17
    D. 3:1.   1 mark

37. In the NRI sector, approximately how many degrees should be there in the central angle?

    A. 36°
    B. 37°
    C. 40°
    D. 42°.   1 mark

38. If the total investment flows from Offshore Funds were to be doubled in the next year and the investment flows from all other sources had remained constant at their existing levels for this year, then what would be the percentage of Offshore Funds investment in the total investment flows into India Bonds in the next year (approximately)?

    A. 27.6%
    B. 32.0%
    C. 25.6%
    D. 30. 0%.   2 marks

English

39. Fill in the blank: The songs in most of the Indian movies can be ignored as they are usually not _____ to the story.

   A. intervened
   B. innate
   C. integral
   D. integrating.   1 mark

40. Fill in the blank: A big part of the population in urban cities is suffering from stress _____ by overwork.

   A. brought
   B. brought on
   C. brought around
   D. brought for.   1 mark

41. Fill in the blank: The boy was too shy to _____ the girl _____.

   A. ask, out
   B. ask, around
   C. take, around
   D. take, away.   1 mark

42. Fill in the blank: He _____ with the fight as his opponent was very _____.

   A. walked in, strong
   B. walked off, weak
   C. won, weak
   D. lost, strong.   1 mark

43. One with an irrational urge to steal in the absence of an economic motive is called

   A. Shoplifter
   B. Kleptomaniac
   C. Drapetomaniac
   D. Pilferer.   1 mark

44. Correct the underlined part of the following sentence: He does not need to explain anything this incident – his behavior <u>is speaking itself</u>.

   A. has been speaking itself
   B. speaks itself
   C. speaks for itself
   D. speaks about itself.   1 mark

45. The quality of unselfish concern for the welfare of others is called

    A. Altruism
    B. Humanism
    C. Egocentrism
    D. Agathism.   1 mark

46. The meaning of the phrase "to pull a rabbit out of hat" is

    A. to pull someone out of danger.
    B. to bring out a problematic situation which was hidden
    C. to do something surprising
    D. to expose hidden enemies.   1 mark

47. Find out the alternative which will replace '?'

Cousin : Extended :: Son : ?

    A. Family
    B. Father
    C. Nuclear
    D. Uncle.   1 mark

48. Find out the alternative which will replace '?'

Meek : Subservient :: Particular : ?

    A. Modest
    B. Furious
    C. Chesty
    D. Fussy.   1 mark

49. Correct the underlined part of the following sentence: In Roman times, getting out of bed on the left side <u>would be thought as bringing bad luck</u>.

    A. was thought to bring bad luck
    B. had been thought as to bring bad luck
    C. was thought bringing bad luck
    D. used to be thought as a bad luck.   1 mark

50. Fill in the blank: The cricket tournament was scheduled to be held in April but was _____ due to general elections.

    A. crossed out
    B. dropped off
    C. put off
    D. turned down.   1 mark

51. Fill in the blank: You _____ the invigilator in yesterday's examination before you borrowed the log table from your classmate.

    A. must inform
    B. ought to have informed
    C. could inform
    D. need to have informed.   1 mark

52. The word which is not the synonym of "to misconceive", is

    A. to misunderstand
    B. to misinterpret
    C. to be amiss
    D. to miscarry.   1 mark

53. Identify the correct sentence

    A. The ship was drowned in the ocean
    B. The professor married with a doctor
    C. The police are coming to catch the thief
    D. One cannot buy all what one likes.   2 marks

54. The first and sixth sentences of a passage are given in the beginning and in the end, respectively. The middle four sentences have been jumbled up. These are labelled as P, Q, R and S. Find out the proper order for the four sentences.

    S1:   As the pressure on land rises, more and more marginal areas in the world are being used for agriculture.

    P: While irrigation may be the most obvious response to drought, it has proved costly and can only benefit a fortunate few.

    Q: Much of this land is located in the arid or semi-arid belts where rain falls irregularly and much of the precious water is soon lost as surface runoff.

    R: There is now increasing interest in a low cost alternative - generally referred to as "water harvesting".

    S: Recent droughts have highlighted the risks to human beings and livestock, which occur when rains falter or fail.

    S6:   Water harvesting is the collection of runoff for productive purposes.

    The proper sequence should be

    A. QRSP
    B. PSQR
    C. QSPR
    D. RPSQ   2 marks

55. The first and sixth sentences of a passage are given in the beginning and in the end, respectively. The middle four sentences have been jumbled up. These are labelled as P, Q, R and S. Find out the proper order for the four sentences.

    S1:   The election of the President of the United States is an indirect voting process.

    P: If both votes result in an absolute majority, the election is over.

    Q: Electors cast votes for the President.

    R: If a majority of electors do not vote for President, the House of Representatives chooses the President.

    S: Citizens cast ballots for a slate of members of the U.S. Electoral College.

    S6:   Presidential elections occur quadrennially on Election Day, which since 1845 has been the Tuesday after the first Monday in November.

    The proper sequence should be

    A. PQRS
    B. SQPR
    C. SPRQ
    D. QRPS   2 marks

56. Fill in the blanks: Freeware is ___(i)___ software that is available for use at no monetary cost. In other words, freeware may be used without ___(ii)___ but may usually not be modified, re-distributed or reverse-engineered without the author's permission. The source code of freeware is typically not ___(iii)___, unlike open source software which are often distributed free of charge. (2 marks).

    A. (i) non-proprietary, (ii) permission, (iii) sold
    B. (i) free, (ii) installing, (iii) written
    C. (i) proprietary, (ii) payment, (iii) available
    D. (i) substandard, (ii) license key, (iii) updated.   2 marks

57. Classify each of the statements given below as Fact (F), Inference (I), Judgment (J).

    I.   Looking at her eyes, it seems that she did not sleep last night.
    II. Her eyes are red and puffy.
    III. She must not be taking proper care of her eyes.

    A. IJI
    B. FFJ
    C. IFJ
    D. JII   2 marks

58. Classify each of the statements given below as Fact (F), Inference (I), Judgment (J).

    I.   Increased frequency of news on crime against women is only a political hoopla.

    II. As the frequency of news on crime against women is increasing, it looks that the criminal mentality is, in general, increasing in the society.

    III. Increased frequency of news on crime against women is only because more cases are being reported nowadays.

    A. FIJ

    B. FIF

    C. FJI

    D. JIJ   2 marks

59. The antonym of "jeopardy" is

    A. peril

    B. danger

    C. secure

    D. deprivation.   1 mark

60. Among the words given below, the one which is not an antonym of "about", is

    A. astir

    B. exactly

    C. just

    D. on the nose.   1 mark

61. Identify the correct sentence.

    A. The student was hopeless to pass

    B. The candidate got an employment in the company

    C. Although it was very hot, but we played Cricket this afternoon

    D. Ramesh takes no care over his work.   2 marks

62. The quality of pizza of a famous local pizza shop has deteriorated over the years. The costs of the pizzas sold by them have increased fourfold in last ten years.

Which of the following would tend to weaken the argument above ?

    A. The volume of sales has increased dramatically over the last ten years.

    B. Unprecedented increase in the cost of the raw material has put severe upward pressures on pizza costs.

    C. It has become difficult to get good quality raw material nowadays.

    D. The average level of consumer prices overall has increased twofold over the last ten years.   2 marks

Read the passage below and answer Question No 63:

Climate change in the form of global warming refers to the rise in average surface temperatures on Earth. The primary cause of climate change is the burning of fossil fuels, such as oil and coal, which emits greenhouse gases into the atmosphere—primarily carbon dioxide. The gases trap heat within the atmosphere, which can have a range of effects on ecosystems, including rising sea levels, severe weather events, and droughts that render landscapes more susceptible to wildfires. Other human activities, such as agriculture and deforestation, also contribute to the proliferation of greenhouse gases. While some quantities of these gases are a naturally occurring and critical part of Earth's temperature control system, the atmospheric concentration of carbon dioxide did not rise above 300 parts per million between the advent of human civilization roughly 10,000 years ago and 1900. Today it is at about 400 ppm, a level not reached in more than 400,000 years.

    I. The atmospheric concentration of carbon dioxide, today,

        i.   has increased from below 300 ppm to 400 ppm in the last 116 years
        ii.   did not change much from the advent of human civilization
        iii. did not change much after 1990.

    II. The primary cause of global warming is/ are

        i.   rising sea levels, severe weather events, and droughts
        ii.   burning of fossil fuels such as oil and coal
        iii. both i and ii above.

    III. Agriculture

        i.   is helping in decreasing the emission of greenhouse gases
        ii.   contributes to the emission of greenhouse gases
        iii. helps in decreasing sea level and earth's temperature.

    63. The correct answers to I, II and III are:

      A. iii, ii, ii respectively
      B. iii, i ii respectively
      C. i, ii, ii respectively
      D. iii, i, i respectively.   3 marks

Logical Reasoning

64. Pointing to a photograph, Mitali said, "He is the son of the only son of my grandfather." How is the man in the photograph related to Mitali?

    A. Brother
    B. Uncle
    C. Son
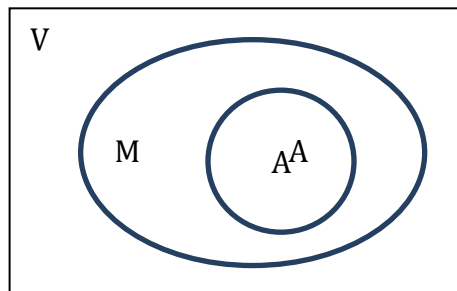   D. Father.   1 mark

65. In a certain code, COMPLAIN is written as CMOLPIAN. How is CIRCULAR written in that code?

    A. ICCRLURA
    B. CRIUCLRA
    C. ICRCLUAR
    D. CRIUCALR   1 mark

66. In the Venn diagram given below, V represents the set of all vehicles, M represents the set of all motorized vehicles, and A represents the set of all automobiles.

A person has reached the following conclusions.

    I.   All automobiles are motorized vehicles.
    II.   All motorized vehicles are automobiles.
    III.   Some automobiles are not vehicles.

Based on the diagram, which of the following are valid conclusions?

    A. I only
    B. III only
    C. I and III
    D. II and III.   1 mark

67. A watch gains 5 seconds in 3 minutes and was set right at 7 AM. In the afternoon of the same day, when the watch indicated quarter past 4 o'clock, the true time is

A. 3:45 PM
B. 4:00 PM
C. 4:30 PM
D. 4:15 PM.   1 mark

68. Today is 3rd November. The day of the week is Monday. This is a leap year. What will be the day of the week on this day after three years?

A. Wednesday
B. Thursday
C. Tuesday
D. Monday   1 mark

69. In the question below, there are three statements followed by two conclusions numbered I and II. You have to take the given statements to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusions logically follows from the given statements.

Statements: *Conclusions:*

1) No fan is a cooler.   (I)   No heater is a tube.
2) Some coolers are heaters.   (II)   Some coolers are tubes.
3) All tubes are fans.

A. Only (I) is a valid conclusion.
B. Only (II) is a valid conclusions.
C. Both (I) and (II) are valid conclusions.
D. Neither (I) not (II) is a valid conclusion.   2 marks

70. The police rounded up Salman, Aamir, Saif and Shahrukh because one of them was suspected of having robbed the local bank. The four suspects made the following statements under intensive questioning.

Salman: Aamir did it.
Aamir :   Shahrukh did it.
Saif: I did not do it.
Shahrukh: Aamir lied when he said that I did it.

If only one of the above statements is true, who did it?

a) Salman
b) Aamir
c) Saif
d) Shahrukh.   3 marks

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